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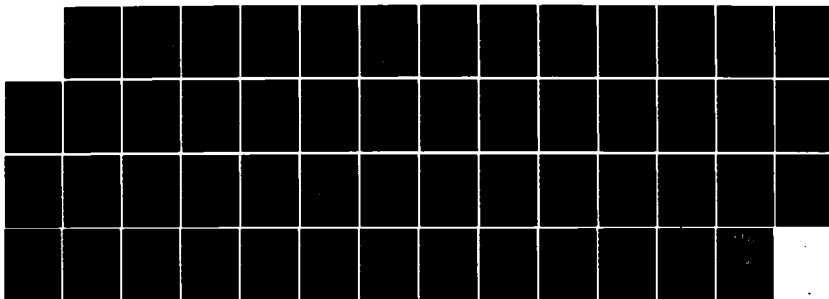
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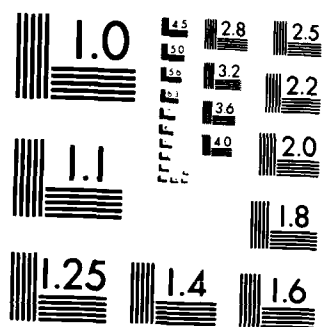
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TECHNICAL REPORT BRL-TR-2628

LIQUID MOMENT ON A FILLED CONING  
CYLINDER DURING SPIN-UP:  
AD HOC MODEL

Nathan Gerber

December 1984

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (bja) A liquid-filled right circular cylinder, coning at constant frequency and small amplitude, impulsively begins spinning with a fixed angular velocity. The ensuing history of the moment, i.e., pressure plus viscous shear, exerted by the liquid on the container is investigated here by computation of the quasi-steady state response of the fluid to the motion of the shell. The <div style="text-align: right;">(continued)</div>		

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partial differential equations of flow are linearized, and a modal analysis (separated variable solution) is applied. The difficulty with endwall boundary conditions which arises from stipulating a modal analysis is avoided by specifying a heuristic boundary condition and then satisfying it approximately. The current procedure for approximating the boundary condition is an improvement over the one employed in previous work (ARBRL-TR-02563) to obtain the pressure moment. Calculations indicate that peaks of overturning moment occur at or near those times when the coning frequency is equal to one of the frequencies of inertial oscillations of the liquid. This phenomenon of resonance extends the results of Stewartson obtained for inviscid perturbations in solid body rotation. Included here also are a discussion of the accuracy of a part of the method and comparisons with output of the inviscid perturbation method and the above-mentioned previous method.

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## I. INTRODUCTION

This report deals with the forces and moments exerted by a liquid payload during spin-up on a spinning and nutating right circular cylinder. The current work stems from the original paper by Stewartson,<sup>1</sup> improved by the viscous correction of Wedemeyer.<sup>2</sup> It represents an extension of the work presented in Reference 3, which treats the particular case of circular angular motion with constant nutational frequency and zero yaw growth. Here we determine the viscous shear contribution to the moment in addition to that of the pressure. The ad hoc model of Murphy,<sup>4</sup> described in Reference 3, is employed to compute the spin-up moment.

In the analysis, two coordinate systems are considered. The first is an inertial system, the unyawed reference frame shown in Figure 1, in which cylindrical polar coordinates\*  $(r, \theta, x)$  and Cartesian coordinates  $(y, z, x)$  are used. The second coordinate system, used to describe the projectile motion, is the  $\tilde{y}, \tilde{z}, \tilde{x}$  non-spinning system that has the  $\tilde{x}$ -axis along the projectile axis of symmetry; the  $\tilde{y}$  and  $\tilde{z}$  axes are omitted from Figure 1 for clarity. The  $x = 0$  and  $\tilde{x} = 0$  values are located at the midplanes of the unyawed and yawed cylinders, respectively. The  $\tilde{x}$ -axis is nutating about the  $x$ -axis with the angle  $K_1(t)$ ; the pivot point lies at the midplane. The components of the projection in the  $y, z$  plane of a unit vector lying on the  $\tilde{x}$ -axis are denoted by  $n_{yE}$  and  $n_{zE}$ , respectively. It is convenient to combine the two components of yaw into a single complex variable

$$\tilde{\xi} = - (n_{yE} + i n_{zE}). \quad (1.1)$$

The nomenclature here is that prescribed in Reference 5 and used in References 3, 6, and 7.

---

\*Definitions of quantities are given in the LIST OF SYMBOLS Section.

1. K. Stewartson, "On the Stability of a Spinning Top Containing Liquid," Journal of Fluid Mechanics, Vol. 5, Part 4, 1959.
2. E. H. Wedemeyer, "Viscous Corrections to Stewartson's Stability Criterion," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Report No. 1287, June 1966. (AD 489687)
3. N. Gorban, "Contribution of Pressure to the Moment During Spin-Up on a Nutating Liquid-Filled Cylinder: Ad Hoc Model," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02563, June 1984. (AL A143678)
4. C. H. Murphy, "Moment Induced by Liquid Payload During Spin-Up Without a Critical Layer," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02581, August 1984. (AD A145716) (See also AIAA 22nd Aerospace Sciences Meeting, Reno, Nevada, AIAA Paper No. 84-0229, January 1984.)

The projectile motion is specified as

$$\tilde{\xi} = K_1 e^{i\phi_1} = K_0 e^{if\dot{\phi}t}, \quad (1.2)$$

where

$$K_1 = K_0 e^{\epsilon\tau\dot{\phi}t}, \quad \phi_1 = \tau\dot{\phi}t, \quad f = (1 - i\epsilon)\tau. \quad (1.3)$$

Here  $t$  is time,  $\dot{\phi}$  (rad/s) is the axial component of the angular velocity of the cylinder;  $\tau$  and  $\epsilon\tau$  are nutational frequency and yaw growth rate, respectively, divided by  $\dot{\phi}$ .  $K_0$  is the magnitude of the yaw at time  $t = 0$ ;  $\phi_1$  is the angular orientation of the  $\tilde{x}$ -axis in the  $y, z, x$  system as shown in Figure 1. Lengths and distances are non-dimensionalized by  $a$ , the cross-sectional radius of the cylinder. The yaw grows when  $\epsilon\tau > 0$ ; however, in this study we apply the restriction  $\epsilon = 0$ .

Computational results will be presented in terms of time histories of non-dimensional moment coefficients. Each history is specified by the three parameters  $Re$ ,  $A \equiv c/a$ , and  $\tau$ , where

$$Re = a^2\dot{\phi}/\nu \quad (1.4)$$

is the Reynolds number and  $\nu$  is the kinematic viscosity of the liquid. The reader is referred to References 3 and 7 for details of the analyses; only the relevant features will be repeated here.

## II. FLOW PROBLEM

### A. Flow Variables.

Here, as in previous work<sup>5,6,7</sup> we assume the yaw to be sufficiently small so that a linearized analysis is applicable, i.e., the flow may be

- 
5. C. H. Murphy, "Angular Motion of a Spinning Projectile with a Viscous Liquid Payload," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-MR-03194, August 1982. (AD A118676) (See also Journal of Guidance, Control, and Dynamics, Vol. 6, July-August 1983, pp. 280-286.)
  6. N. Gerber, R. Sedney, and J. M. Bartos, "Pressure Moment on a Liquid-Filled Projectile: Solid Body Rotation," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02422, October 1982. (AD A120567)
  7. N. Gerber and R. Sedney, "Moment on a Liquid-Filled Spinning and Nutating Projectile: Solid Body Rotation," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02470, February 1983. (AD A125332)

considered the sum of a known basic unperturbed axisymmetric flow and a 3-D perturbation flow. The basic flow employed here is the Wedemeyer spin-up model<sup>8</sup> (see Eqs. (2.2) and (2.3) in Reference 3) which is obtained by finite difference calculations. Thus,

$$u = U(r, \dot{\phi}t) - K_0 \dot{u}^*(r, \theta, x, \dot{\phi}t) \quad (2.1a)$$

$$v = V(r, \dot{\phi}t) - K_0 \dot{v}^*(r, \theta, x, \dot{\phi}t) \quad (2.1b)$$

$$w = W(r, x, \dot{\phi}t) - K_0 \dot{w}^*(r, \theta, x, \dot{\phi}t) \quad (2.1c)$$

$$p = P(r, \dot{\phi}t) - K_0 \dot{p}^*(r, \theta, x, \dot{\phi}t). \quad (2.1d)$$

The  $u, v, w$  are velocity components in the radial, azimuthal, and axial directions, respectively;  $U, V, W$  are the corresponding velocity components of the basic flow. The velocity components of the perturbed flow are  $\dot{u}^*, \dot{v}^*,$  and  $\dot{w}^*$ . The quantity  $p$  is pressure,  $P$  is the pressure of the basic flow, and  $\dot{p}^*$  is the perturbation pressure. Velocity is non-dimensionalized by  $a\dot{\phi}$  and pressure by  $\rho a^2 \dot{\phi}^2$ , where  $\rho$  is the density of the liquid. The angular motion prescribed by Eq. (1.2) leads to boundary conditions on the perturbed velocity components at the sidewall and endwalls with a  $t, \theta$  dependence of  $e^{i(\tau\dot{\phi}t-\theta)}$ .

#### B. Assumptions of Model.

The present model assumes the following form for the perturbed flow variables:

$$\dot{u}^* = \text{Real} [\underline{u}(r, x; \dot{\phi}t) \exp \{i(\tau\dot{\phi}t-\theta)\}] \quad (2.2a)$$

$$\dot{v}^* = \text{Real} [\underline{v}(r, x; \dot{\phi}t) \exp \{i(\tau\dot{\phi}t-\theta)\}] \quad (2.2b)$$

$$\dot{w}^* = \text{Real} [\underline{w}(r, x; \dot{\phi}t) \exp \{i(\tau\dot{\phi}t-\theta)\}] \quad (2.2c)$$

$$\dot{p}^* = \text{Real} [\underline{p}(r, x; \dot{\phi}t) \exp \{i(\tau\dot{\phi}t-\theta)\}]. \quad (2.2d)$$

---

8. E. H. Wedemeyer, "The Unsteady Flow Within a Spinning Cylinder," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Report No. 1225, October 1963. (AD A431846) (See also Journal of Fluid Mechanics, Vol. 20, Part 3, 1964, pp. 383-399.)

The  $\underline{u}$ ,  $\underline{v}$ ,  $\underline{w}$ , and  $\underline{p}$  are complex functions; the time,  $t$ , enters them only as a parameter due to the "quasi-steady" assumption (discussed at the end of Section II.A in Reference 3). The resulting problem in  $\underline{u}$ ,  $\underline{v}$ ,  $\underline{w}$ , and  $\underline{p}$  is formally time-independent, implying that there are no transients in the lag between the motion of the cylinder and the corresponding response of the flow. The solution of Eq. (2.2) would be most valid late in the spin-up history of the fluid, and its applicability would be expected to decrease as  $t$  decreases.

The differential equations for  $\underline{u}$ ,  $\underline{v}$ ,  $\underline{w}$ , and  $\underline{p}$  are given by Eq. (2.9) in Reference 3, the axis and sidewall boundary conditions by Eqs. (2.10) and (2.11) in the same report.  $V(r)$  and  $\partial V/\partial r$  occur in these, but  $U$  and  $W$  are absent.

The following ad hoc endwall boundary condition is applied in the absence of a rationally derived boundary condition:

$$L(\underline{w}) \equiv (1 \mp \delta c_E \partial/\partial x) \underline{w} = i (1 - \tau) r \quad \text{at } x = \pm A. \quad (2.3)$$

It is discussed briefly in the last two paragraphs of Chapter I of Reference 3. This condition will not be satisfied exactly by the solution but only to within the accuracy of a least squares fit. The complex "thickness"  $\delta c_E$  is found as follows:

$$\alpha_E = 2^{-1/2} \text{Re}_E^{1/2} (1 - i) (3 - \tau)^{1/2} \quad (2.4a)$$

$$\beta_E = 2^{-1/2} \text{Re}_E^{1/2} (1 + i) (1 + \tau)^{1/2} \quad (2.4b)$$

$$\delta c_E = \left[ \frac{1}{2\alpha_E} \left(1 - \frac{2}{1-\tau}\right) + \frac{1}{2\beta_E} \left(1 + \frac{2}{1-\tau}\right) \right], \quad (2.4c)$$

with the restriction  $(3 - \tau) \geq 0$ . The "effective" Reynolds number used here is given by

$$\text{Re}_E(t) = \left[ 2 \int_0^1 V(r; \dot{\phi}t) dr \right] \text{Re}. \quad (2.5)$$

For solid-body rotation, with  $V = r$ , the bracketed term is equal to unity, and  $\text{Re}_E = \text{Re}$ ; while the fluid is spinning up,  $V < r$  and  $\text{Re}_E < \text{Re}$ .

### C. Flow Solution.

The flow solution is expressed as

$$\underline{u} = \hat{u}_0(r) x + \sum_{k=1}^{KF} \hat{u}_k(r) \sin \lambda_k x + \sum_{j=1}^{NJ} d_j \bar{u}_j(r) \sin \mu_j x \quad (2.6a)$$

$$\underline{v} = \hat{v}_0(r) x + \sum_{k=1}^{KF} \hat{v}_k(r) \sin \lambda_k x + \sum_{j=1}^{NJ} d_j \bar{v}_j(r) \sin \mu_j x \quad (2.6b)$$

$$\underline{w} = \hat{w}_0(r) - \sum_{k=1}^{KF} \hat{w}_k(r) \cos \lambda_k x - \sum_{j=1}^{NJ} d_j \bar{w}_j(r) \cos \mu_j x \quad (2.6c)$$

$$\underline{p} = \hat{p}_0(r) x + \sum_{k=1}^{KF} \hat{p}_k(r) \sin \lambda_k x + \sum_{j=1}^{NJ} d_j \bar{p}_j(r) \sin \mu_j x, \quad (2.6d)$$

where  $\hat{u}_0$ ,  $\hat{u}_k$ ,  $\bar{u}_j$ , etc. satisfy Eq. (2.23) of Reference 3.

The  $\lambda_k$ 's are solutions to the functional equation (see Eq. (31) of Reference 6)

$$\cos \lambda_k A + \lambda_k \delta c_E \sin \lambda_k A = 0, \quad (2.7)$$

so that every  $\hat{w}_k(r) \cos \lambda_k x$  term satisfies the homogeneous endwall condition  $L(\hat{w}_k \cos \lambda_k x) = 0$  at  $x = \pm A$ , where the operator  $L$  is defined in Eq. (2.3).

At  $r = 1$ ,  $\bar{u}_j = \bar{v}_j = \bar{w}_j = 0$ , so that the  $\mu_j$ 's are eigenvalues of Eq. (2.23) of Reference 3 with  $\lambda_k$  replaced by  $\mu_j$ . (The normalization of the corresponding eigenfunctions is described in Chapter III.C of Reference 9.) Thus, the partial solution taken from Eq. (2.6),

---

1. J. W. Dettmer, Jr., L. Barber, and S. Demey, "Oscillation of a Fluid in a Rotating Body," *Journal of Applied Physics*, Vol. 41, No. 1, p. 100, 1970.

$$\underline{u}_n = \hat{u}_0(r) x + \sum_k \hat{u}_k(r) \sin \lambda_k x, \quad \text{etc.}, \quad (2.8)$$

satisfies the sidewall conditions (Eq. (2.11) of Reference 3):

$$\underline{u}(r=1) = \underline{u}_n(r=1) = -i(1-\tau)x \quad (2.9a)$$

$$\underline{v}(r=1) = \underline{v}_n(r=1) = -\{V_r(r=1) - \tau\}x \quad (2.9b)$$

$$\underline{w}(r=1) = \underline{w}_n(r=1) = i(1-\tau). \quad (2.9c)$$

Finally, the partial solution

$$\underline{u}_p = \hat{u}_0(r) x + \sum_j d_j \bar{u}_j(r) \sin \mu_j x \quad (2.10a)$$

$$\underline{w}_p = \hat{w}_0(r) + \sum_j d_j \bar{w}_j(r) \cos \mu_j x, \quad \text{etc.}, \quad (2.10b)$$

is left to satisfy the non-homogeneous endwall boundary condition of Eq. (2.3). The  $\underline{w}_p$  of Eq. (2.10b) cannot satisfy Eq. (2.3) identically; the complex  $d_j$ 's are therefore determined so as to minimize in a least squares sense the error over the interval  $0 \leq r \leq 1$ .

#### D. Operational Procedure.

The current operational procedure differs from the one employed in Reference 3. The new method was suggested by C. H. Murphy. The first step is to calculate the complex eigenvalues  $\mu_j$  and the eigenfunctions  $\bar{u}_j, \bar{v}_j, \bar{w}_j, \bar{p}_j$  as before for  $j = 1, \dots, N_d$ .

The next step is to solve the differential equations for  $\hat{u}_0, \hat{v}_0, \hat{w}_0, \hat{p}_0$  (Eq. (2.23) of Reference 3, with  $\lambda_k = 0$  and  $r_p = 1$ ). The orthonormalization process of Reference 9 is employed to obtain three linearly independent continuous solutions  $(\hat{u}_i, \hat{v}_i, \hat{w}_i, \hat{p}_i, \hat{v}_i', \hat{w}_i')$ , where  $i = 1, 2, 3$ , by applying the following three independent sets of boundary conditions at  $r = 1$ :

$$\hat{u}_1(1) = 1, \quad \hat{v}_1(1) = 0, \quad \hat{w}_1(1) = 0 \quad (2.11a)$$

$$\hat{u}_2(1) = 0, \quad \hat{v}_2(1) = 1, \quad \hat{w}_2(1) = 0 \quad (2.11b)$$

$$\hat{U}_3(1) = -\frac{(1-\tau)}{(1+\tau)}, \quad \hat{V}_3(1) = \frac{i(1-\tau)}{(1+\tau)}, \quad \hat{W}_3(1) = 1. \quad (2.11c)$$

The third set, Eq. (2.11c), is  $[i(1-\tau)]^{-1}$  times the boundary values of the particular solution for solid body rotation. In providing a known limiting case, these latter numbers prove to be helpful in checking the program and assessing the accuracy of the procedure. Finally,

$$\begin{aligned} \hat{u}_0 &= k_1 \hat{U}_1 + k_2 \hat{U}_2 + k_3 \hat{U}_3 \\ \hat{v}_0 &= k_1 \hat{V}_1 + k_2 \hat{V}_2 + k_3 \hat{V}_3 \\ \hat{w}_0 &= k_1 \hat{W}_1 + k_2 \hat{W}_2 + k_3 \hat{W}_3, \quad \text{etc,} \end{aligned} \quad (2.12)$$

where  $k_1$ ,  $k_2$ , and  $k_3$  are constants to be determined.

We apply the same condition on  $\hat{w}_0(1)$  as on  $\underline{w}(1)$ ; namely, Eq. (2.9c):

$$\hat{w}_0(1) = i(1-\tau). \quad (2.13)$$

Then, by application of Eq. (2.11),

$$k_3 = i(1-\tau) \quad (2.14)$$

and

$$\hat{w}_0(r) = k_1 \hat{W}_1(r) + k_2 \hat{W}_2(r) + i(1-\tau) \hat{W}_3(r). \quad (2.15)$$

Upon substitution from Eq. (2.15) and Eq. (2.10b), the endwall ad hoc boundary condition at  $x = A$  becomes

$$R(r) \equiv \psi(r) - \sum_{j=1}^{NJ+2} e_j \bar{w}_j(r) = 0, \quad (2.16)$$

where

$$e_j = (\cos \mu_j A + \delta c_E \mu_j \sin \mu_j A) d_j \quad (j = 1, 2, \dots, NJ) \quad (2.17a)$$

$$e_{NJ+1} = k_1, \quad e_{NJ+2} = k_2 \quad (2.17b)$$

$$\bar{w}_{NJ+1} = -\hat{w}_1(r) \quad (2.17c)$$

$$\bar{w}_{NJ+2} = -\hat{w}_2(r) \quad (2.17d)$$

$$\psi(r) = i(1-\tau)[\hat{w}_3(r) - r]. \quad (2.17e)$$

The error,  $R(r)$ , will not be identically zero. We seek to optimize the fit of the solution to the boundary condition by determining a set of  $e_j$ 's which minimizes the error integral

$$g(e_1, \dots, e_{NJ+2}) \equiv \int_0^1 |R(r)|^2 dr. \quad (2.18)$$

The procedure for calculating the  $e_j$ 's is presented in Appendix A. There are now  $NJ+2$  functions and constants with which to minimize the error, as compared with  $NJ$  functions and constants in the method of Reference 3. This additional flexibility is expected, generally, to produce an improvement in accuracy in satisfying the endwall boundary condition. The measure of the relative error adopted here is given by

$$Er = [g / \int_0^1 |\psi(r)|^2 dr]^{1/2}. \quad (2.19)$$

The constants  $k_1$ ,  $k_2$ , and  $k_3$  are now evaluated from Eqs. (2.17b) and (2.14).

Finally, the differential equations (Eq. (2.23) of Reference 3 with  $\epsilon_p = 0$ ) are solved for  $\hat{u}_k$ ,  $\hat{v}_k$ ,  $\hat{w}_k$ ,  $\hat{p}_k$  with the boundary conditions at  $r = 1$  obtained by subtracting  $\hat{u}_0(1)x$ ,  $\hat{v}_0(1)x$ ,  $\hat{w}_0(1)$  from the conditions of Eq. (2.9). Thus for  $k \neq 0$ ,

$$\hat{u}_k(1) = -[\hat{u}_0(1) + i(1-\tau)]b_k \quad (2.20a)$$

$$\hat{v}_k(1) = -[\hat{v}_0(1) + v_r(1) - \tau]b_k \quad (2.20b)$$

$$\hat{w}_k(1) = 0, \quad (2.20c)$$

where

$$b_k = \frac{(2/\lambda_k)^2 [1 + (\lambda_k \delta c_E)^2] \sin \lambda_k A}{A [1 + (\lambda_k \delta c_E)^2] - \delta c_E}, \quad (2.21)$$



given by Eq. (2.29) of Reference 3. The complete solution is now available to compute forces and moments.

### III. LIQUID MOMENTS

#### A. Sidewall Moment.

Definitions of sidewall pressure and shear moment coefficients are stated in Eq. (5.6) of Reference 7.  $M_{L\tilde{Z}L}$  is the moment about the (transverse)  $\tilde{z}$ -axis exerted by the liquid on the sidewall of the container:

$$M_{L\tilde{Z}L}/(2\pi\rho a^4 c \dot{\phi}^2 \tau K_1) = C_{(LSM)PL} \sin \tau\dot{\phi}t + C_{(LIM)PL} \cos \tau\dot{\phi}t + \quad (3.1)$$

$$C_{(LSM)VL} \sin \tau\dot{\phi}t + C_{(LIM)VL} \cos \tau\dot{\phi}t.$$

The LSM and LIM designate side and in-plane moments, respectively; i.e., the moments tending to change the yaw angle and nutational frequency, respectively. P indicates pressure, V indicates viscous wall shear, and the final L (lateral) designates sidewall.

The boundary layer assumptions (p. 20 of Reference 7) are retained in the derivation of the shear moment; thus,  $\partial \tilde{u}^*/\partial r$ ,  $\partial \tilde{u}^*/\partial \theta$ , and  $\partial \tilde{u}^*/\partial x$  at  $r = 1$  are dropped. For spin-up, additional order of magnitude assumptions are made. While terms of  $O(K_0/Re)$  are kept, terms of  $O(K_0/Re^{3/2})$  are omitted; thus,  $U$ ,  $\partial U/\partial r$ ,  $\partial W/\partial r$ , and  $\partial W/\partial z$  are dropped. The term  $[V_r(1) - 1]$  appears in the moment coefficient formula, and it is also neglected. Because of the boundary layer in the perturbed flow, it is expected that  $|\partial \tilde{v}^*/\partial r|$  and  $|\partial \tilde{w}^*/\partial r| \gg [\partial V/\partial r - V/r]$  at  $r = 1$  except at very early time when the theory is not applied.

We replace Eq. (5.7) of Reference 7 with the following set of formulas applicable to spin-up for the present circumstances ( $\epsilon = \ell = 0$ ), where integrands are evaluated at  $r = 1$ :

$$C_{(LSM)PL} = (2\tau A)^{-1} \text{Real} \left[ -i \int_{-A}^A x (p+x) dx \right] \quad (3.2a)$$

$$C_{(LIM)PL} = (2\tau A)^{-1} \text{Imag} \left[ -i \int_{-A}^A x (p+x) dx \right] \quad (3.2b)$$

$$C_{(LSM)VL} = \frac{1}{2\tau A \text{Re}} \text{Real} \left[ -i \int_{-A}^A \left\{ \frac{\partial w}{\partial r} - i x \frac{\partial v}{\partial r} - i x^2 \left( \frac{\partial v}{\partial r} - \tau \right) \right\} dx \right] \quad (3.2c)$$

$$C_{(LIM)VL} = \frac{1}{2\tau A \text{Re}} \text{Imag} \left[ -i \int_{-A}^A \left\{ \frac{\partial w}{\partial r} - i x \frac{\partial v}{\partial r} - i x^2 \left( \frac{\partial v}{\partial r} - \tau \right) \right\} dx \right]. \quad (3.2d)$$

The  $\partial v / \partial r$  term replaces unity in Eq. (5.7c) of Reference 7 as a consequence of the sidewall condition on  $\tilde{v}$  (Eq. (2.6b) in Reference 3). (The third term of the integral in Eqs. (3.2c) and (3.2d) has been retained here and in Reference 7 even through other terms of the same order in Re have been dropped in the boundary layer approximation.) The variables  $p$ ,  $v$ , and  $w$  are evaluated by the formulas in Eq. (2.6). Detailed expressions for the integrals in Eqs. (3.2a) and (3.2b) are given in Eqs. (3.8), (3.9), and (3.10) of Reference 3.

#### B. Endwall Moment.

$M_{L\tilde{z}E}$  is the moment about the (transverse)  $\tilde{z}$ -axis exerted by the liquid on the endwalls of the container. We restate Eq. (5.12) of Reference 7:

$$M_{L\tilde{z}E} / (2\pi \rho a^4 \dot{\phi}^2 \tau K_1) = C_{(LSM)PE} \sin \tau \dot{\phi} t + C_{(LIM)PE} \cos \tau \dot{\phi} t + \\ C_{(LSM)VE} \sin \tau \dot{\phi} t + C_{(LIM)VE} \cos \tau \dot{\phi} t. \quad (3.3)$$

The endwall pressure moment coefficients are given by

$$C_{(LSM)PE} = [1/(\tau A)] \text{Real} \left[ i \int_0^1 r^2 \{ p(r,A) + A v^2/r \} dr \right] \quad (3.4a)$$

$$C_{(LIM)PE} = [1/(\tau A)] \text{Imag} \left[ i \int_0^1 r^2 \{ p(r,A) + A v^2/r \} dr \right], \quad (3.4b)$$

which are the same as those obtained from Eqs. (3.12) - (3.18) in Reference 3.

For the endwall shear

$$C_{(LSM)VE} = - [1/(\tau \text{ Re})] \text{Real} \left[ \int_0^1 r \left\{ \frac{\partial (\underline{v} - i \underline{u})}{\partial x} \right\}_{x=A} dr \right] \quad (3.5a)$$

$$C_{(LIM)VE} = - [1/(\tau \text{ Re})] \text{Imag} \left[ \int_0^1 r \left\{ \frac{\partial (\underline{v} - i \underline{u})}{\partial x} \right\}_{x=A} dr \right], \quad (3.5b)$$

the same as Eqs. (5.13c) and (5.13d) of Reference 7. The treatment of the term  $\partial (\underline{v} - i \underline{u}) / \partial x$  parallels that in Section V of Reference 7, where it is shown that the second term dominates the right-hand side of Eq. (5.19). Analogously, here

$$\int_0^1 r \left\{ \frac{\partial (\underline{v} - i \underline{u})}{\partial x} \right\}_{x=A} dr \approx B_E \int_0^1 r [-A(1-2\tau + v_r(r)) + i r v_r(r)] dr, \quad (3.6)$$

where

$$r v_r = (\hat{v}_0 - i \hat{u}_0) A + \sum_{k=1}^{KF} (\hat{v}_k - i \hat{u}_k) \sin \lambda_k A + \sum_{j=1}^{NJ} d_j (\bar{v}_j - i \bar{u}_j) \sin \mu_j A. \quad (3.7)$$

### C. Total Moment Coefficient.

The total side moment coefficients due to pressure and shear stress, respectively, are

$$C_{(LSM)P} = C_{(LSM)PL} + C_{(LSM)PE} \quad (3.8a)$$

$$C_{(LSM)V} = C_{(LSM)VL} + C_{(LSM)VE}. \quad (3.8b)$$

The total side moment coefficient is

$$C_{(LSM)} = C_{(LSM)P} + C_{(LSM)V}. \quad (3.9)$$

We now define the following side moment coefficient:

$$C_L \equiv C_{LSM} \tau. \quad (3.10)$$

For given cylinder, liquid, and spin,  $C_L$  is proportional to side moment, regardless of the perturbing frequency.

#### IV. COMPUTATIONS

##### A. Results.

We present side moment coefficient histories for the same four combinations of  $Re$  and  $A$  that were considered in Reference 3. These cover a Reynolds number range of approximately  $5 \times 10^3$  to  $2 \times 10^6$ . Here we plot  $C_L$  instead of  $C_{LSM}$ . There is an advantage to this representation in that relative amplitudes of moment are immediately apparent for all perturbing frequencies. Again, the results are compared with inviscid perturbation calculations produced by the method of Murphy.<sup>4</sup>

In Figure 2,  $C_L$  histories are shown for five nutational frequencies:  $\tau = 0.04, 0.05, 0.09, 0.12$ , and  $0.14$ . These were computed with the Wedemeyer laminar Ekman layer spin-up flow model.<sup>8</sup> The results are qualitatively similar to those of Figure 2 in Reference 3. The  $\tau = 0.04$  curve asymptotically approaches its maximum with increasing  $\phi t$ , indicating a resonance for spun-up liquid. All the other curves have peaks demonstrating resonance with the  $(k, n) = (3, 1)$  natural oscillation mode of the liquid.\* The  $\tau = 0.12$  and  $0.14$  curves have additional peaks (right-hand peaks) produced by resonance with the  $(k, n) = (5, 2)$  mode. There appears to be a maximum overturning moment that the casing can experience, produced by a particular nutational frequency at a particular time. In this instance the maximizing frequency is very likely close to  $\tau = 0.09$  and the time close to  $\phi t = 1200$ . For  $\tau = 0.12$ , the amplitude of the  $(k, n) = (5, 2)$  mode is small compared to that of the  $(k, n) = (3, 1)$  mode; however, for  $\tau = 0.14$  the two amplitudes are comparable.

The inviscid perturbation results are also shown for  $\tau = 0.04, 0.05$ , and  $0.09$ ; for  $\tau = 0.12$  and  $0.14$  inviscid results are unobtainable because of the presence of the "critical layer" (discussed in References 10 and 11), occurring when  $V/r = \tau$  at some  $r$ . The relative positions of the curves are

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\*The reader is referred to References 9 and 10 for descriptions of the oscillation modes of a liquid cylinder.

10. R. Tedney and N. Gerber, "Oscillations of a Liquid in a Rotating Cylinder: Part II. Spin-Up," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, AEPRL-TR-02489, May 1988. (AD A190011)
11. R. Tedney and N. Gerber, "A Study of the Critical Layer in a Rotating Liquid Payload," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, AEPRL-TR-02582, August 1984. (See also AIAA Paper No. 84-0342, January, 1984.)

similar to those in Figure 2 of Reference 3, but the relative discrepancies are now larger, for a discrepancy in viscous shear moment is now added to that in pressure moment. However, the predictions of times of peak moments still agree to within about 0.07 s.

Figure 3 deals with a case of smaller spin-up time (spin-up time defined in Chapter I of Reference 12); therefore, the interval of interest applies to smaller  $\dot{\phi}t$ . The Ekman layer is taken to be laminar in the spin-up flow model. In Figure 4 the eigenfrequency histories of five oscillation modes during spin-up show the times when resonances with the coning frequency  $\tau = 0.15$  occur, i.e., when  $C_R = 0.15$ . The time of occurrence of the peak obtained by the present method,  $\dot{\phi}t \approx 225$ , indicates a resonance with the  $(k, n) = (3, 1)$  mode. The inviscid perturbation method fails to show this resonance; possibly the method deteriorates with decreasing time shortly before it breaks down due to the critical layer. The inviscid computation shows a prominent peak at  $\dot{\phi}t \approx 250$  in Figure 3 where the frequencies (computed with inviscid perturbations) of the  $(k, n) = (5, 2)$  and  $(7, 3)$  modes are approximately equal to 0.15. The present calculation shows no indication of resonance at this time, though a jog appears in  $C_{(LSM)p}$  in Figure 3 of Reference 3.\*

The present computation does not extend below  $\dot{\phi}t \approx 200$  because of problems (described in Chapter II.C of Reference 3) which arise at "small" times. These are manifested in the difficulty encountered in obtaining certain of the  $\mu_j$ 's. While this difficulty is not necessarily insuperable, the required labor can at times render the effort infeasible.

In Figure 5, moment coefficient histories are shown for  $Re = 1.85 \times 10^6$  for two aspect ratios differing by 2.55%. The turbulent Ekman layer was employed in the spin-up model here. The sensitivity of side moment to  $c/a$  is borne out by the two widely differing responses to disturbances of the same frequency. Both cases can be plotted on the same graph since the times of significant variation in the two sets of curves differ markedly ( $\dot{\phi}t \sim 10,000$  for  $A = 4.973$  and  $\dot{\phi}t \sim 40,000$  for  $A = 5.100$ ).

We first consider the  $c/a = 5.100$  case. Figure 4 in Reference 3 demonstrates the agreement between the present and inviscid perturbation methods for the pressure contribution alone. Discrepancies between the two outputs are noticeably increased when shear moment is included, particularly near the times of occurrence of the peaks. In the present method, the viscous shear increases the overturning moment by as much as 25% at  $\dot{\phi}t = 42,000$ ; whereas, the inviscid perturbation method generally yields a slight decrease. However, the two curves approach each other as  $\dot{\phi}t$  increases, reaching a limiting value

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\*The present computation in Figure 3 of Reference 3 is incorrect in the region of  $\dot{\phi}t$  where the influence exhibits no peak there.

12. A. Lebeck and R. Garber, "Viscous Effects in the Wedemeyer Model of Spin-Up from Rest," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ABRRL-TR-22194, June 1984. (AD A154600)

of  $C_L = 2.95 \times 10^{-4}$ . The predictions of times of peak moments differ between the two methods by approximately 1.6 s for  $C_{L,SM}$  (Figure 4 of Reference 3) and by approximately 1.8 s for  $C_L$ .

For  $c/a = 4.973$  the addition of shear moment does not affect the inviscid output qualitatively, as indicated by inspection of present Figure 5 and Figure 5 of Reference 3. However, it drastically changes the picture for the present method; the  $C_L$  curve has a minimum with negative value as before, but it also has a peak immediately afterward. The peaks for both aspect ratios indicate resonance with the  $(k, n) = (5, 1)$  mode of the liquid.

### B. Accuracy of Results.

Confidence in the numerical results requires some estimate of their accuracy. A thorough-going error study for wide ranges of physical parameters is not feasible; we confine our attention to the cases treated in Section IV.A. There are at least twelve parameters in the spin-up and perturbed Navier-Stokes calculations alone that control the accuracy of the numerical procedures. Accuracy as used here is quite different from validation of the theory. There are, in fact, no experimental data to compare with the present theory.

In all cases we computed spin-up profiles with  $r$ -interval  $\Delta r = 1/200$  and time interval  $\Delta(\dot{t})$  small enough so that five or fewer iterations were required for convergence. For the perturbation equations, we took  $\Delta r = 1/1000$  and performed no less than fifty orthonormalizations. We used as many as thirty terms of the power series to evaluate the solution to the perturbation equations at  $r = 0.001$  where the numerical integration began, and we used seven terms ( $b_k \sin \lambda_k x$ ) in the biorthogonal expansion of the function  $x$ . In all iterative processes, we required convergence at least to five decimal places.

In this problem an additional factor, apart from those introduced by the numerical operations, must be considered; namely, the error  $E_r$  of Eq. (2.19) resulting from approximating  $\psi(r)$  by a finite series,  $\sum_{j=1}^N e_j w_j(r)$ , in

Eqs. (2.16) and (2.17). This approximation is made in order to minimize the overall error resulting from failure of the solution to satisfy the endwall boundary condition identically. In the procedure of Reference 3, designated "Method I" here, we used  $N_1 \bar{w}_j$ 's; whereas, in the present procedure, designated "Method II," we employ  $(N_1 + 2) \bar{w}_j$ 's, the first  $N_1$  of which are identical to those of Method I. Whenever possible, we set  $N_1 = 6$ , but it is not always possible to do so.

Figures 6, 7, 8 and 9 present comparisons of errors for Methods I and II. In both methods,  $\psi(r) = 0$  and all  $e_j$ 's = 0 at solid body rotation, so that in principle  $E_r(\dot{t} = 0)$  is indeterminate. Results indicate a general trend in which the error decreases as time decreases from  $\dot{t} = \infty$ , reaches a minimum, and

then increases, sometimes at a rapid rate, as  $t \rightarrow 0$ . In Figure 6, Method II yields a considerably smaller error than Method I, being always less than 1/2 percent; the same result holds for  $\tau = 0.04$  and  $0.09$ . One would expect a more accurate approximation from Method II because there are  $NJ + 2$ , as compared to  $NJ$ , functions with which to approximate  $\psi(r)$ . This is true in most of our cases. However, the error of Method II can also become very large, as shown in Figure 7. We note that  $\psi(r)$  and the functions with which it is fitted generally have sharp dips or spikes near  $r = 1$  because of the sidewall boundary layer. Furthermore, at early time when a critical layer is present, these functions often have sharp oscillations in the critical layer. These two factors should account in part for the large error of Method II at early time.

Method II produces a considerably smaller error than Method I for the case in Figure 8 and for the  $c/a = 5.100$  case in Figure 9. For the  $c/a = 4.973$  case in Figure 9 the region of resonance is at early time, and the two methods have comparable errors. In all the error plots except those in Figure 7, six eigenvalues  $\mu_j$  and corresponding eigenfunctions were used. In the plot for  $\tau = 0.12$ , due to the difficulty of computing other eigenvalues, only the  $n = 1, 2, 3, 4$  modes were used for  $\dot{\phi}t \geq 1000$  and the  $n = 1, 2, 4$  modes for  $\dot{\phi}t < 1000$ .

The eigenvalues  $\mu_j$ , together with their associated eigenfunctions,  $\bar{w}(r)$ , are important parameters of the solution, not only in the number used but also in the choice of eigenvalues and functions when a choice is available. At solid body rotation the radial oscillation modes are identified according to the increasing value of the real part of the eigenvalue and designated as  $n = 1, 2$ , etc. The oscillation modes retain their identity with decreasing time, but eventually their history has to be traced in small time decrements since some of the Real ( $\mu_n$ ) vs  $t$  curves cross.

Experience indicates that the most accurate calculations are obtained when the lowest possible modes are employed in the approximation to  $\psi(r)$ ; e.g., if  $NJ = 4$ , the  $j = 1, 2, 3, 4$  in Eq. (2.6) should correspond to  $n = 1, 2, 3, 4$ . Table I provides an illustration. To three figures, the ( $n = 1, 2, 3, 4$ ) combination gives the same  $C_L$  values as the six-mode calculation, which presumably is the most accurate choice; and the  $C_L$  from the ( $n = 1, 2, 3$ ) combination differs from these by less than 3%. It should be noted, however, that agreement between the four- and six-mode approximations is not always as good as that obtained in this particular case. The other combinations give values of  $C_L$  that differ by at least 10 and 13% from 0.0204. In this instance the sensitivity to choice of modes is more notable than sensitivity to number of modes. The third and fourth combinations in Table I can arise in the actual moment calculation. Under certain circumstances, the iteration process may converge to an eigenvalue other than the one being sought if the initial guess is not sufficiently close to the final answer. This phenomenon is discussed on page 21 of Reference 3. In the case shown in Table 2, the two determinations of  $C_L$  differ by about 3%.

Figures 10, 11, and 12 compare the  $C_L$ 's obtained by Methods I and II. For the  $Re = 49,772$  and  $Re = 1.85 \times 10^6$  cases there is general agreement between the outputs of the two methods even though the  $Ir$ 's disagree (Figures 6 and 9). Thus,  $Ir$  is not a sensitive indicator of the effect on  $C_L$  of the

method employed. For the  $Re = 4974$  case, the discrepancies are appreciable; e.g., at  $\dot{\phi}t = 400$  the  $C_L$  of Method I is less than half of the  $C_L$  of Method II. In light of the errors plotted in Figure 8, we conclude that Method II yields the more accurate value of  $C_L$  in this particular case.

TABLE 1. EFFECT OF RADIAL MODES ON ERROR AND MOMENT COEFFICIENT:  
 $Re = 4974$ ,  $A = 3.30$ ,  $\tau = 0.15$ ,  $\dot{\phi}t = 220$ .

<u>Mode (n) Combinations</u>	<u>Er</u>	<u><math>C_L</math></u>
1, 2, 3, 4, 5, 6	0.14	0.02044
1, 2, 3, 4	0.58	0.02043
1, 3, 4, 5	3.65	0.02268
2, 3, 4, 6	10.89	0.01674
1, 2, 3	1.38	0.02051

TABLE 2.  $Er$  AND  $C_L$  FOR TWO SETS OF MODES:  $Re = 39,772$ ,  
 $A = 3.12$ ,  $\tau = 0.14$ ,  $\dot{\phi}t = 1000$ .

<u>Mode (n) Combinations</u>	<u>Er</u>	<u><math>C_L</math></u>
1, 2, 3, 4, 5, 6	0.97	0.000637
1, 2, 3, 4	4.14	0.000585

## V. SUMMARY

The primary mechanism for producing unstable motion of projectiles with fully spun-up liquid payload is resonance between the coning motion of the projectile and inertial oscillations of the contained liquid. Resonance also occurs early in the flight during spin-up, but the transient natures of the fluid flow and projectile angular motion make prediction of instability more difficult than for solid body rotation. This report describes a procedure, based on linearized flow, for calculating the side moment exerted by the liquid on the casing during spin-up for the special case of constant coning frequency and zero yaw growth. Here the moment due to shear forces is included in addition to that due to pressure forces. The heuristic approach of Murphy<sup>4</sup> is used which makes crucial assumptions regarding time-dependence



of the flow and the endwall boundary condition. Because of the specialized angular motion the present treatment cannot simulate actual flight, although it can simulate performable gyroscope experiments. The validity of this method is expected to be strongest at late time and diminish with decreasing time.

Calculations were performed for four combinations of  $Re$  and  $A$  and a variety of nutational frequencies. The peaks of the  $C_L$  vs  $\dot{\phi}t$  curves indicate transient coincidence of certain inertial frequencies of the liquid with the frequency of coning. For a given container, payload, and spin, there is a finite frequency which maximizes the side moment during the course of spin-up. The duration of a side moment surge, as well as its amplitude, will undoubtedly be a factor in determining onset of instability in a free-flight situation. Comparison of output with results of the inviscid perturbation method shows qualitative agreement at late time but differing indications of resonance at early time, as, for example, in Figure 3.

In the current analysis an ad hoc correction was made to the endwall boundary condition which is an extension of the correction for the solid body rotation problem. A subsequent refinement should be the inclusion of the Ekman layers in the basic flow. This step is necessary in order to have a rational approximation to the solution. It is not expected to be easy to achieve.

#### ACKNOWLEDGEMENTS

The author is pleased to acknowledge the help of Ms. Joan M. Bartos, who programmed and performed the moment calculations. Appreciation is extended to Dr. Raymond Sedney for his overall guidance. The moment data for inviscid perturbation were obtained from calculations performed with a program prepared by Mr. James W. Bradley.

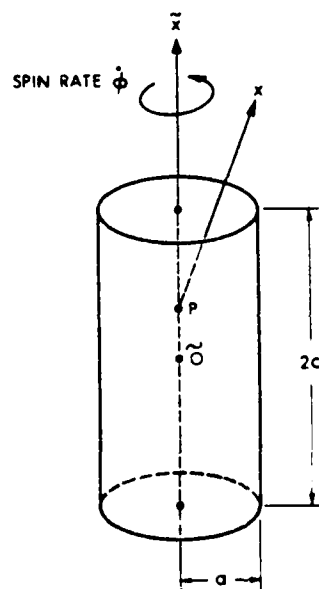
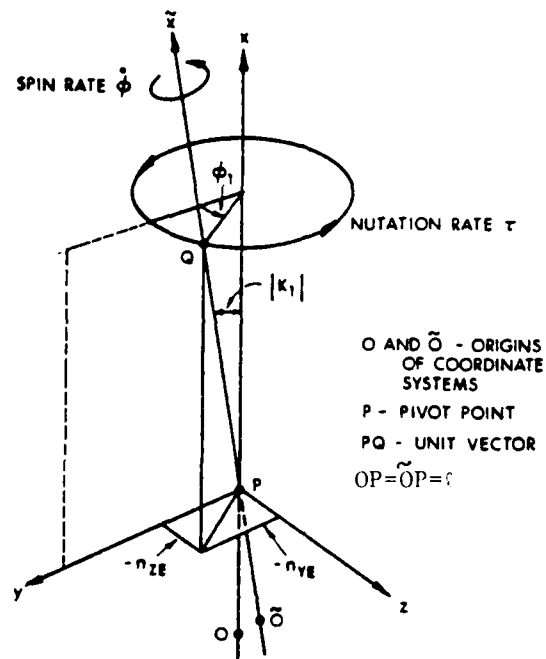


Figure 1. Diagrams of Coordinates and Cylinder.

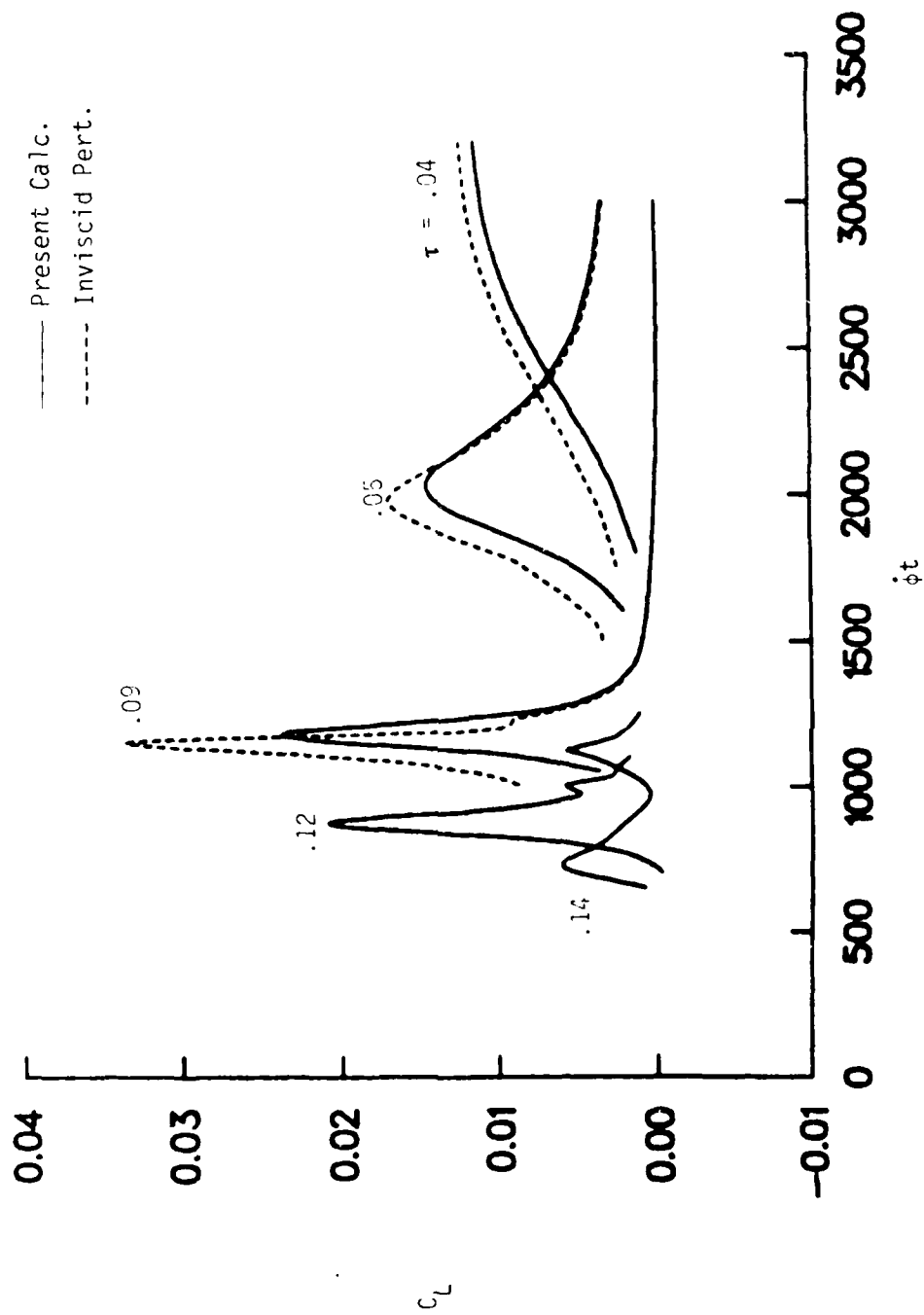


Figure 2. Side Moment Coefficient Histories for Five Nutational Frequencies,  $Re = 39772$ ,  $A = 3.12$ ,  $\dot{\phi} = 754$  rad/s.

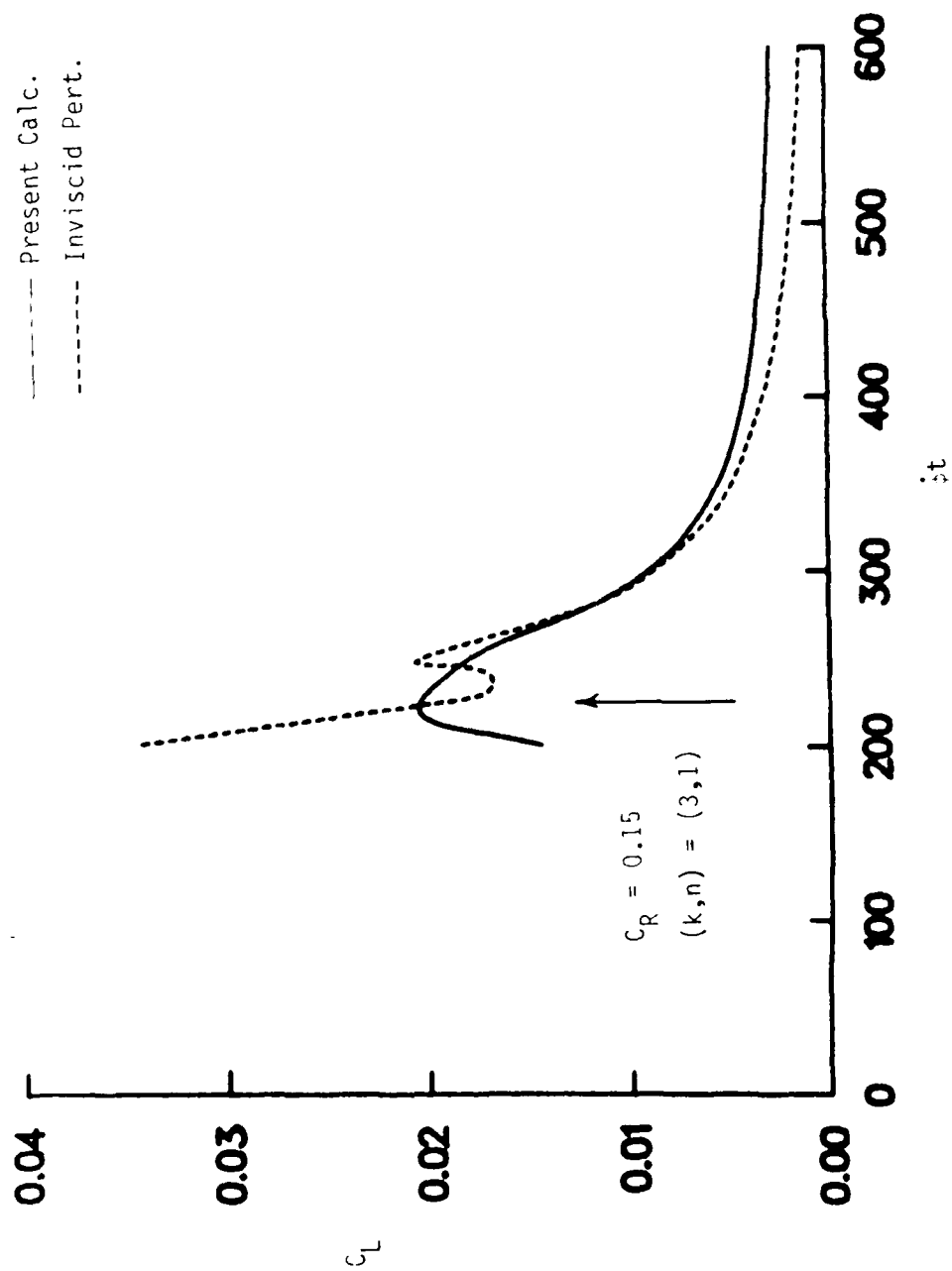


Figure 3. Side Moment Coefficient History,  $Re = 4974$ ,  $A = 3.30$ ,  $\dot{\theta} = 8937$  rad/s,  $\tau = 0.15$ .

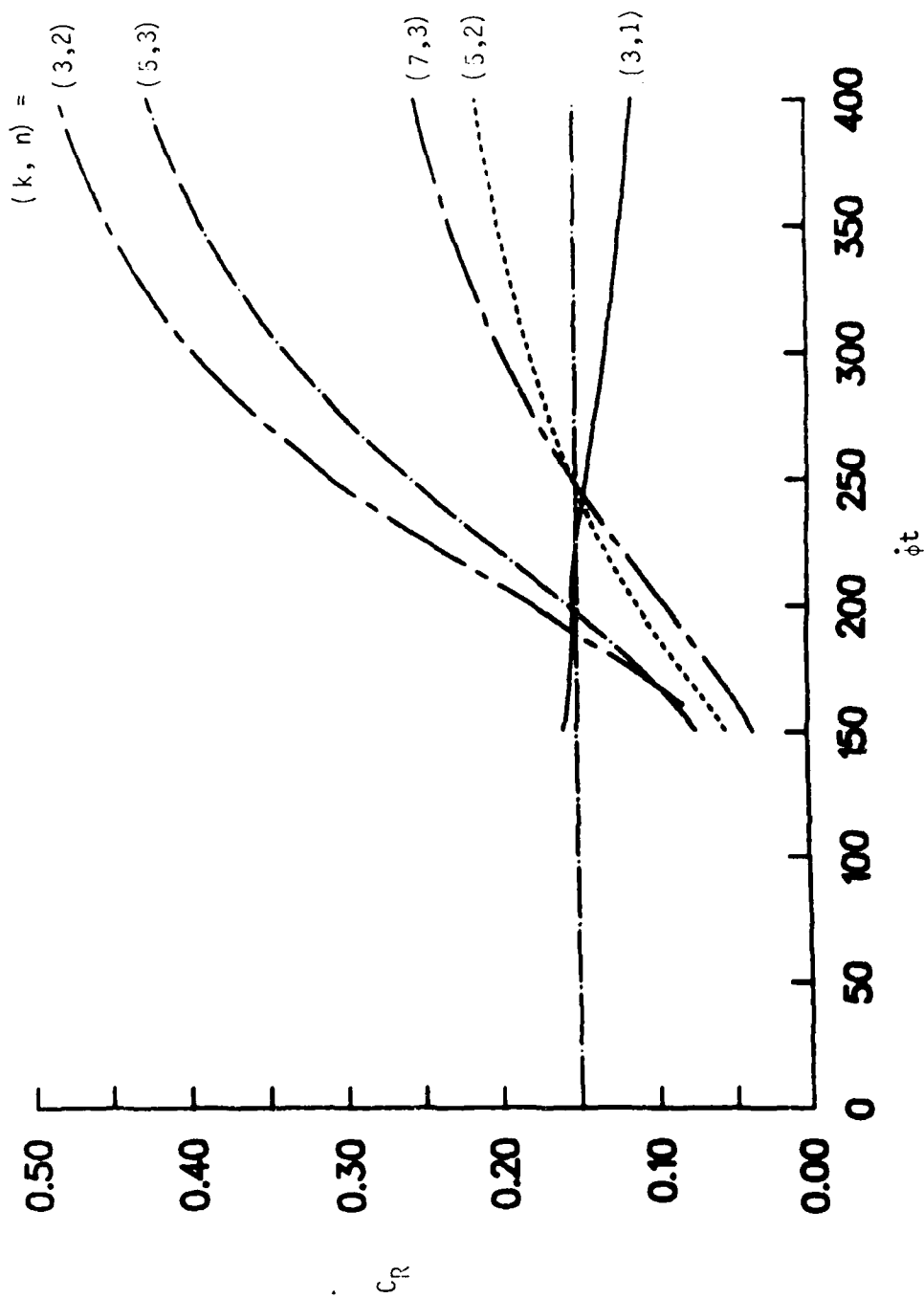


Figure 4. Spin-Up Eigenfrequency Histories for Five Modes,  $Re = 4974$ ,  $A = 3.30$ .

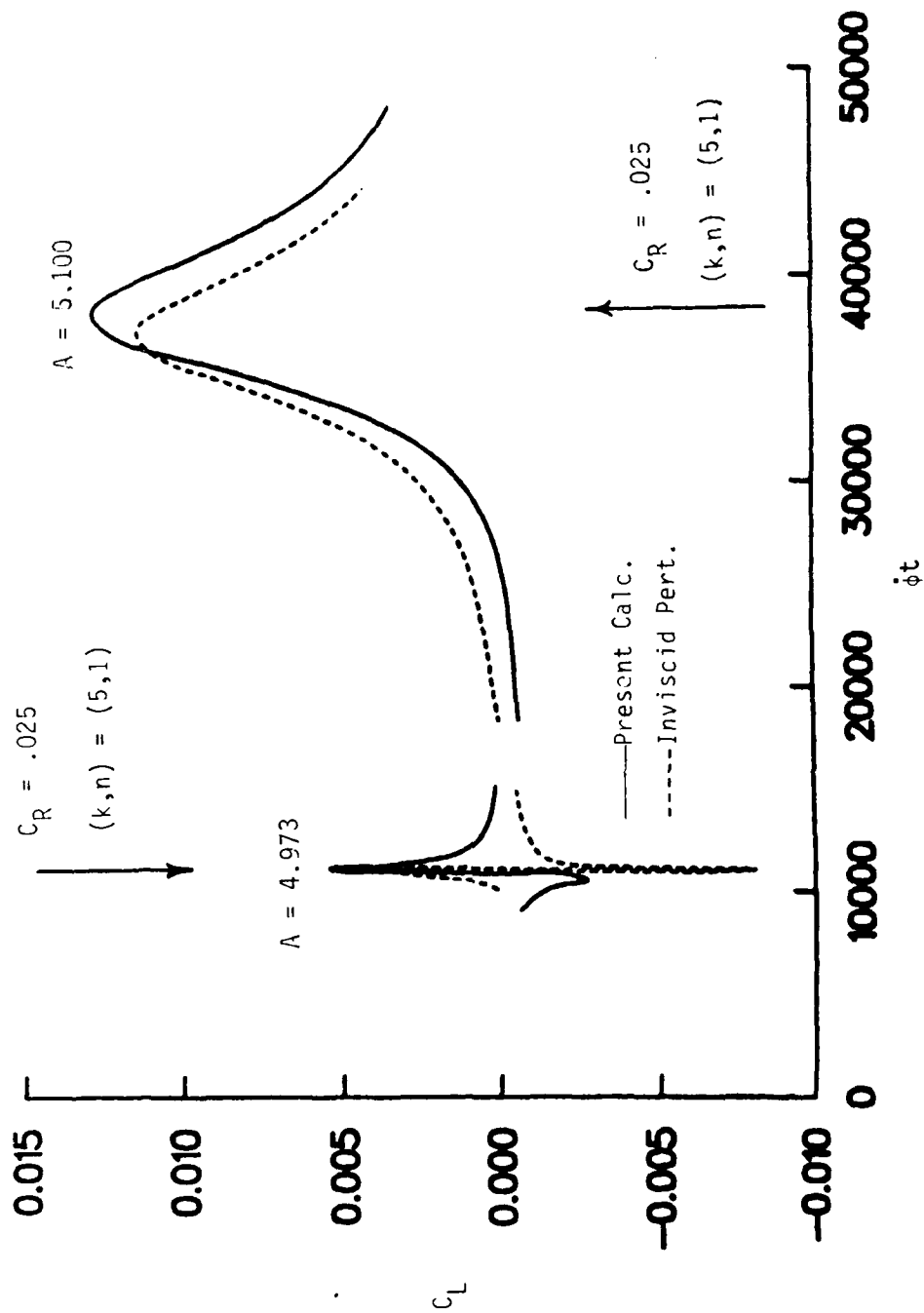


Figure 5. Side Moment Coefficient Histories for Two Aspect Ratios,  
 $Re = 1.85 \times 10^6$ ,  $\dot{\phi} = 641$  rad/s,  $\tau = 0.025$ .

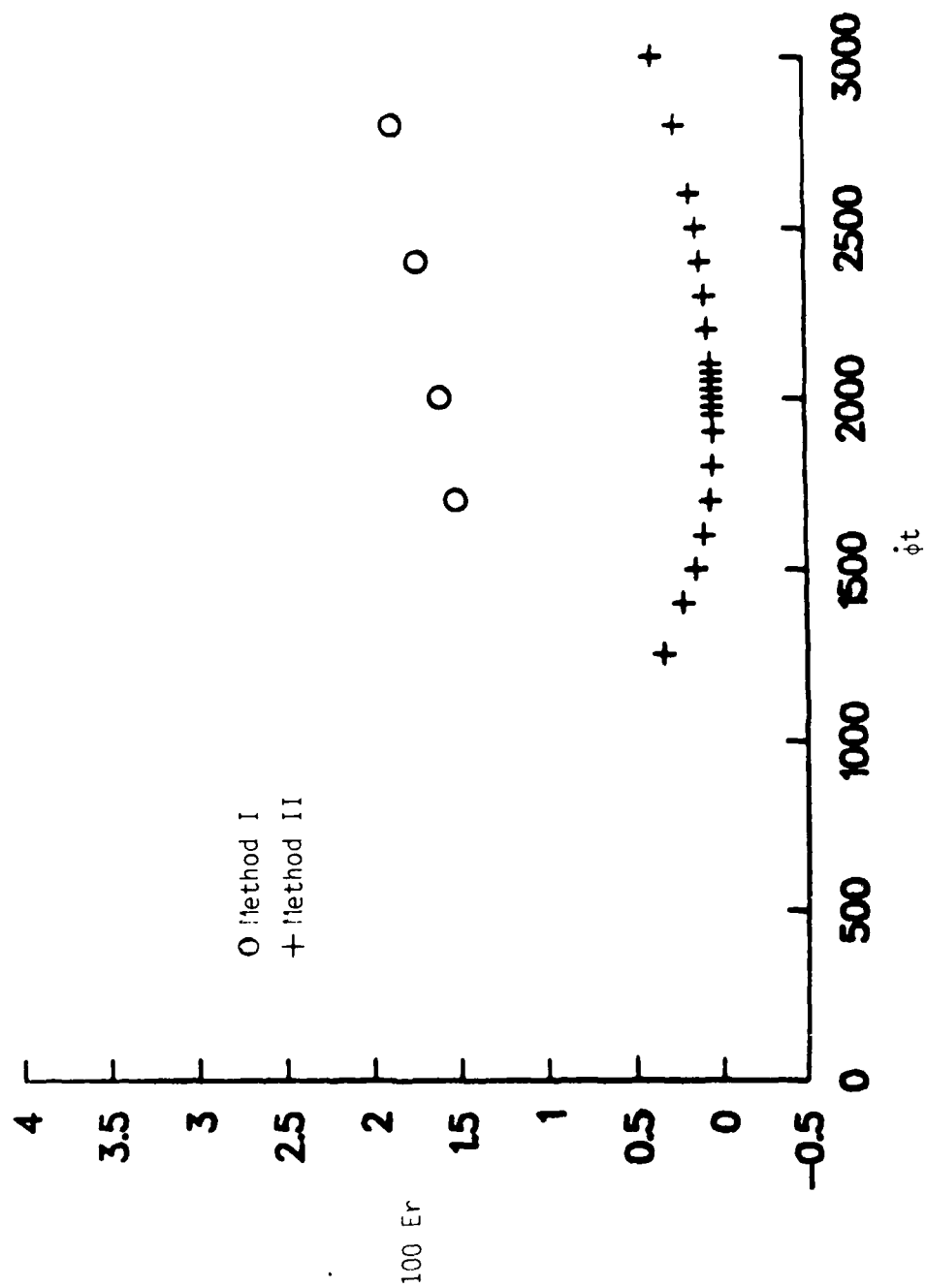


Figure 6. Comparison Between Percentage Errors of Methods I and II,  
 $Re = 39772$ ,  $A = 3.12$ ,  $\tau = 0.05$ .

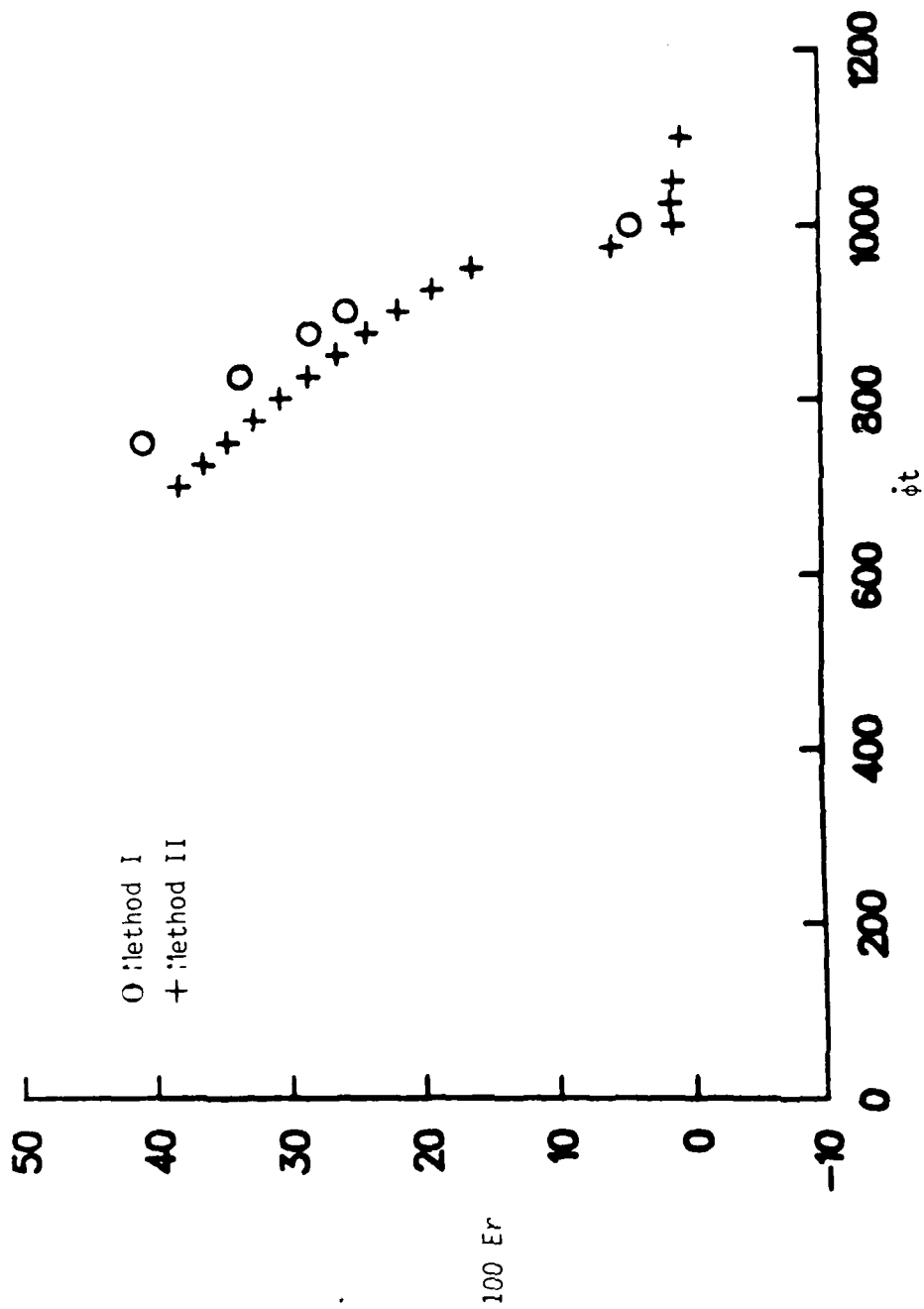


Figure 7. Comparison Between Percentage Errors of Methods I and II,  
 $Re = 39772$ ,  $A = 3.12$ ,  $\tau = 0.12$ .



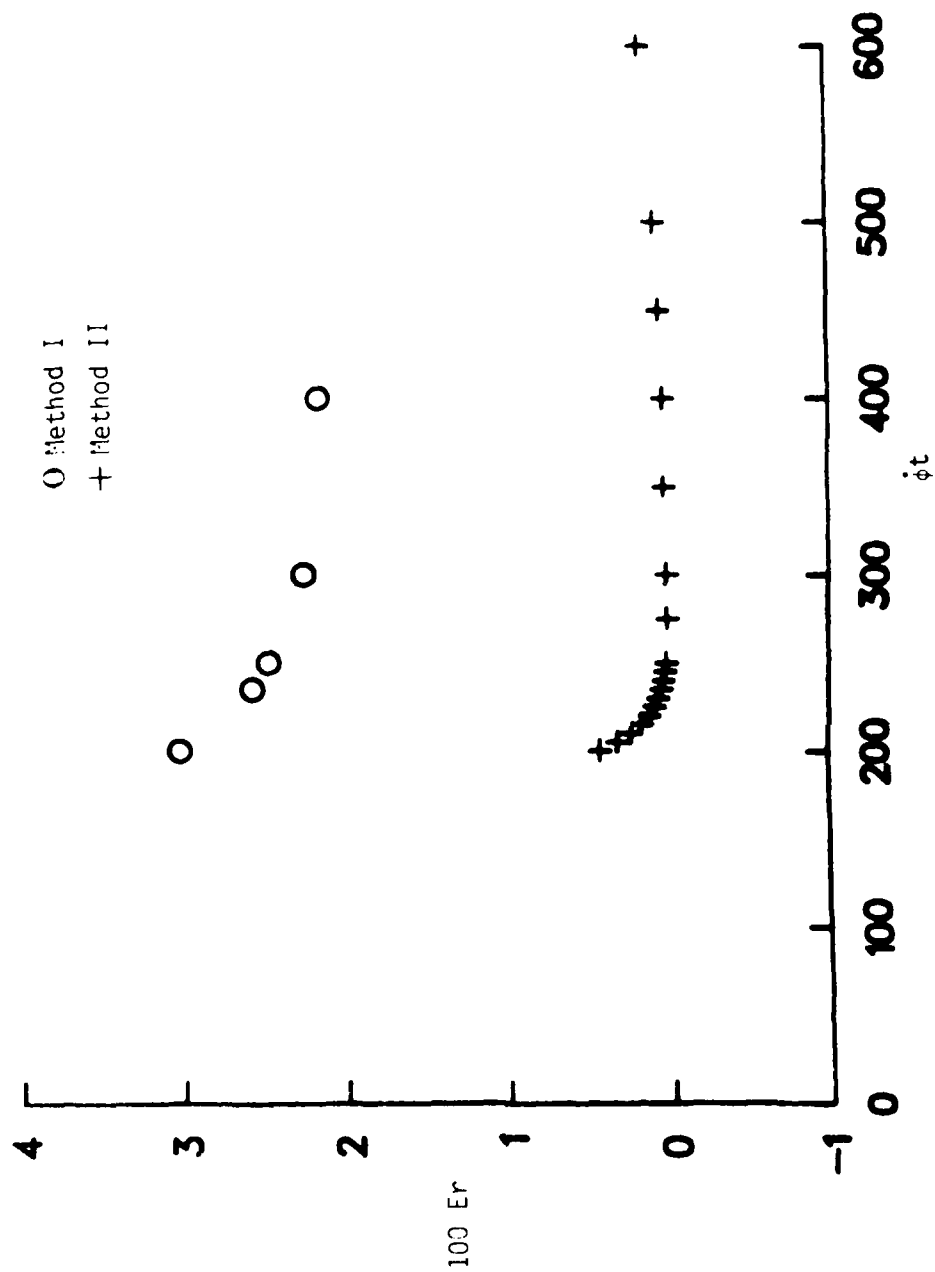


Figure 8. Comparison Between Percentage Errors of Methods I and II,  
 $Re = 4974$ ,  $A = 3.30$ ,  $\tau = 0.15$ .

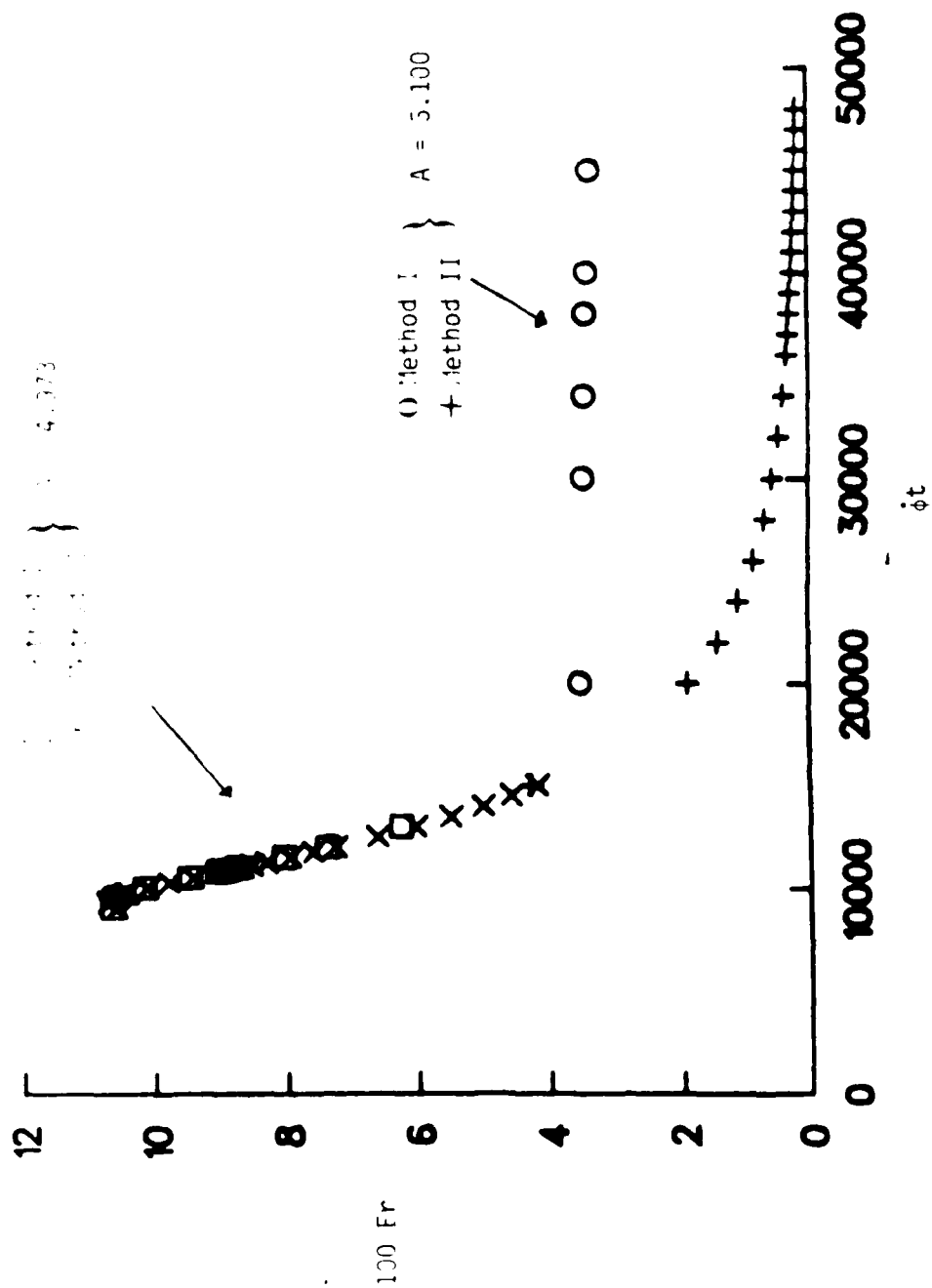


Figure 9. Comparison Between Percentage Errors of Methods I and II,  
 $Re = 1.85 \times 10^6$ ,  $\tau = 0.025$ .

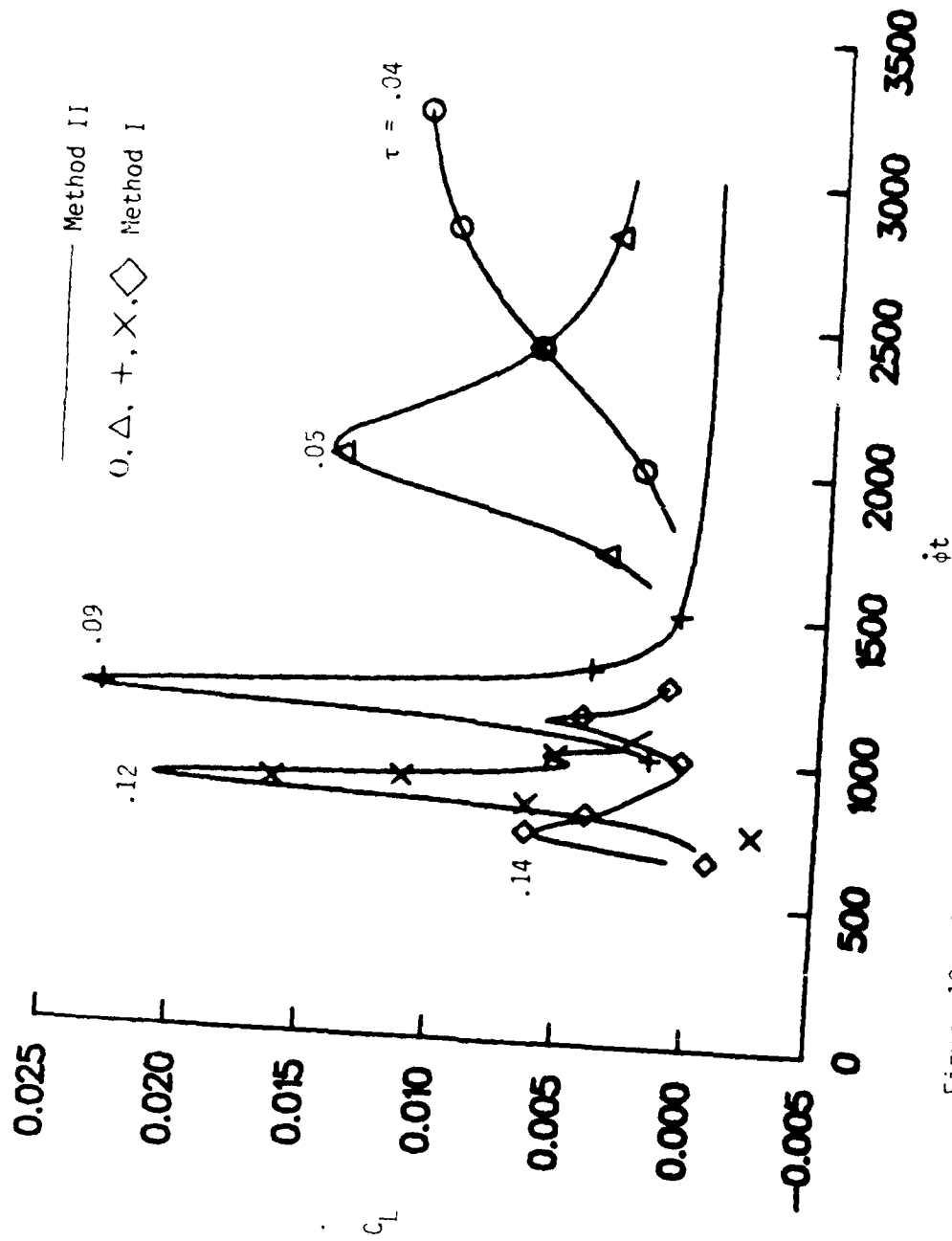


Figure 10. Comparison Between Side Moment Coefficients of Methods I and II,  
 $Re = 39772$ ,  $A = 3.12$ .

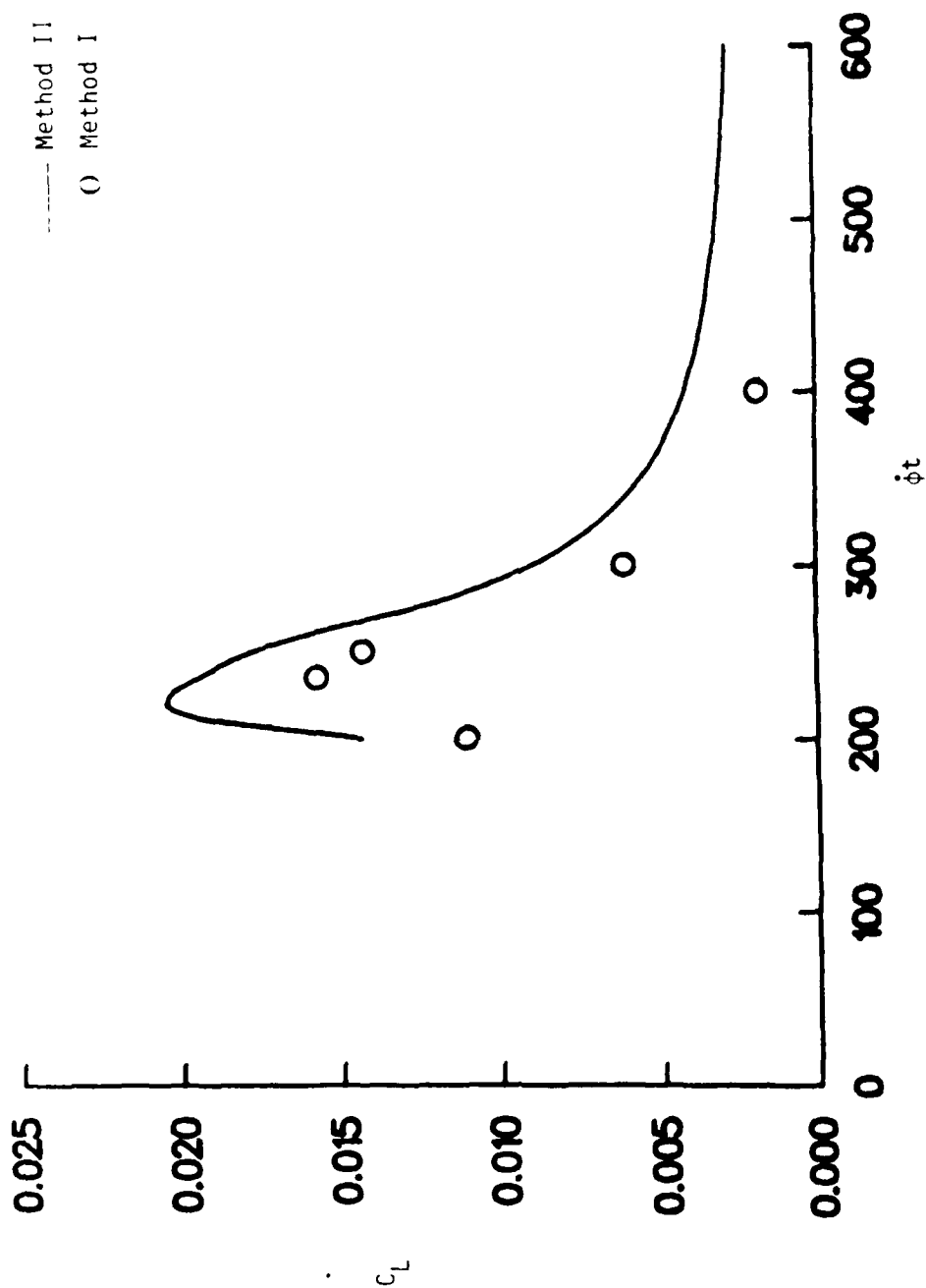


Figure 11. Comparison Between Side Moment Coefficients of Methods I and II,  
 $Re = 4974$ ,  $A = 3.30$ ,  $\tau = 0.15$ .

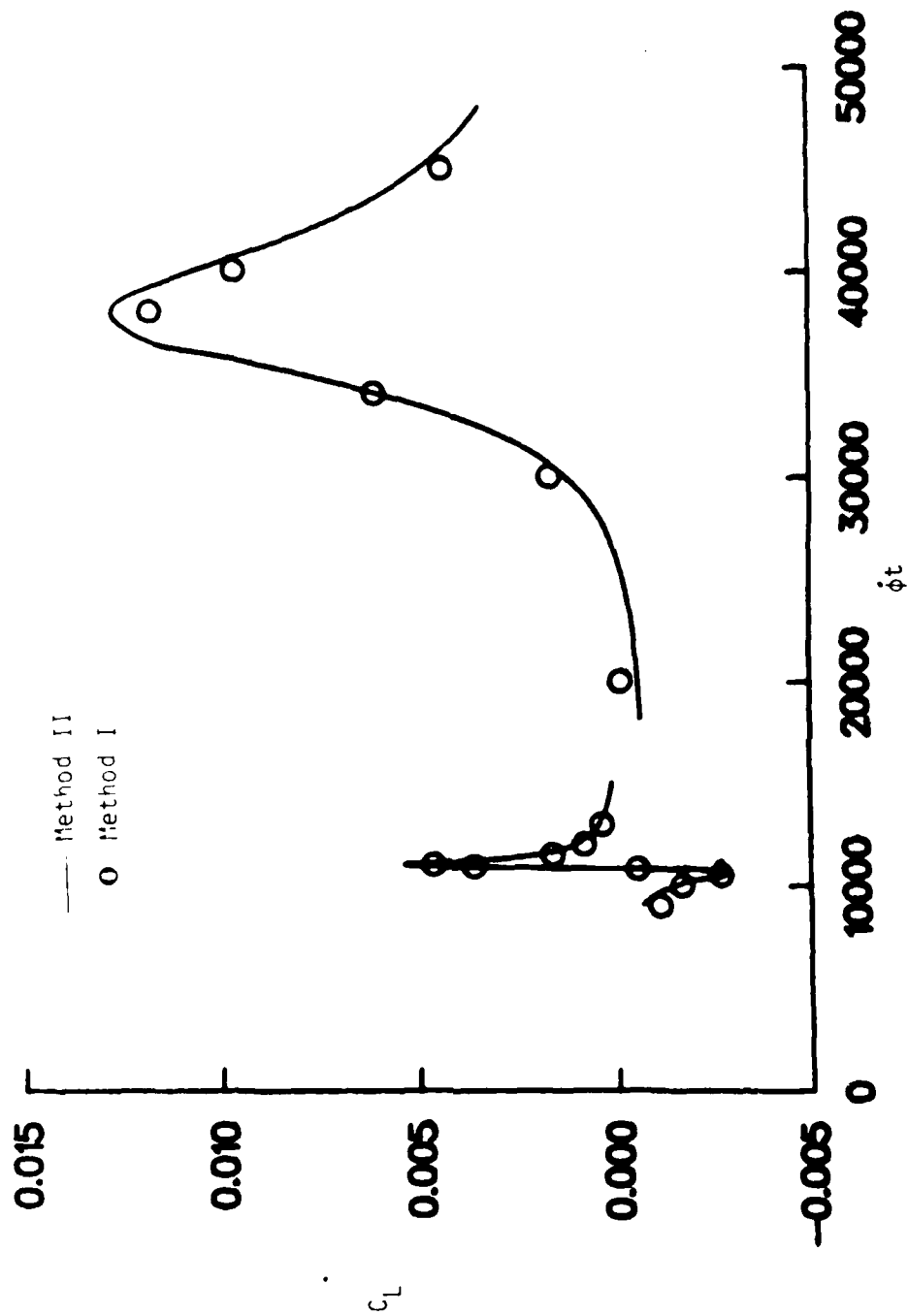


Figure 12. Comparison Between Side Moment Coefficients of Methods I and II,  
 $Re = 1.85 \times 10^6$ ,  $\tau = 0.025$ .

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APPENDIX A  
CALCULATION OF  $e_j$ 's

# APPENDIX A. CALCULATION OF $e_j$ 's

Eq. (2.16) is restated:

$$R(r) = \psi(r) - \sum_{j=1}^{NJ+2} e_j \bar{w}_j(r). \quad (A.1)$$

We now define the following elements:

$$s_n = \int_0^1 \psi(r) \times \text{conj} [\bar{w}_n(r)] dr \quad n = 1, \dots, NJ + 2 \quad (A.2)$$

$$s_{n,j} = \int_0^1 \bar{w}_j \times \text{conj} [\bar{w}_n] dr \quad n, j = 1, \dots, NJ + 2, \quad (A.3)$$

where  $\text{conj} [\bar{w}_n]$  is the complex conjugate of  $\bar{w}_n$ .

The  $e_j$ 's which minimize the error integral  $g$  of Eq. (2.18) are found by solving the following system of linear complex equations:

$$\sum_{j=1}^{NJ+2} s_{nj} e_j = s_n \quad n = 1, \dots, NJ + 2. \quad (A.4)$$



# LIST OF SYMBOLS

$a$	cross-sectional radius of cylinder, Figure 1 [cm]
$A$	$\pm c/a$ , aspect ratio of cylinder
$b_k$	biorthogonal coefficients for series expansion of $x$ in interval $-A \leq x \leq A$ , Eq. (2.21)
$c$	half-height of cylinder [cm]
$C_L$	$C_{LSM}$ $\tau$ , a side moment coefficient
$C_{(LIM)PE}$	endwall pressure in-plane moment coefficient, Eqs. (3.3) and (3.4b)
$C_{(LIM)PL}$	sidewall pressure in-plane moment coefficient, Eqs. (3.1) and (3.2b)
$C_{(LIM)VE}$	endwall viscous shear in-plane moment coefficient, Eqs. (3.3) and (3.5b)
$C_{(LIM)VL}$	sidewall viscous shear in-plane moment coefficient, Eqs. (3.1) and (3.2d)
$C_{LSM}$	total side moment coefficient, Eq. (3.9)
$C_{(LSM)P}$	pressure side moment coefficient, Eq. (3.8a)
$C_{(LSM)PE}$	endwall pressure side moment coefficient, Eqs. (3.3) and (3.4a)
$C_{(LSM)PL}$	sidewall pressure side moment coefficient, Eqs. (3.1) and (3.2a)
$C_{(LSM)V}$	viscous shear side moment coefficient, Eq. (3.8b)
$C_{(LSM)VE}$	endwall viscous shear side moment coefficient, Eqs. (3.3) and (3.5a)
$C_{(LSM)VL}$	sidewall viscous shear side moment coefficient, Eqs. (3.1) and (3.2c)
$C_R$	natural oscillation frequency of rotating liquid/ $\frac{1}{2}$

# LIST OF SYMBOLS (Continued)

$d_j$	coefficient in series given by the 3rd terms on right-hand sides of Eqs. (2.6a) - (2.6d)
$e_j$	coefficient defined in Eq. (2.17a)
$Er$	relative error measure defined in Eq. (2.19)
$f$	$\pm (1 - i\varepsilon)\tau$ , complex representation of angular motion, Eq. (1.3)
$g(e_j)$	error integral defined in Eq. (2.18)
$k$	index of axial eigenfunction and eigenvalue
$k_1, k_2, k_3$	coefficients of combination of linearly independent solutions, (Eq. (2.12))
$KF$	index of final term of $\sin \lambda_k x$ and $\cos \lambda_k x$ in Eq. (2.6)
$K_0$	yaw amplitude at time $t = 0$
$K_1$	$K_0 e^{\varepsilon \tau \dot{\phi} t}$ , yaw amplitude at time $t$ , Eq. (1.3)
$\tilde{x}$	nondimensional $x$ (and $\tilde{x}$ ) coordinate of pivot point
$M_{L\tilde{L}}, M_{L\tilde{E}}$	sidewall and endwall contributions, respectively, to liquid moment about $\tilde{z}$ -axis [ $g \text{ cm}^2/\text{s}^2$ ]
$n$	index of radial mode for eigenfrequency, $C_R$
$n_{y\tilde{L}}, n_{z\tilde{L}}$	components in the $y, z$ plane of a unit vector lying on the $\tilde{x}$ -axis
$N$	number of terms in the series given by the 3rd terms of the right-hand sides of Eqs. (2.6)

# LIST OF SYMBOLS (Continued)

$p$	pressure/ $(\rho a^2 \dot{\phi}^2)$
$p(r, x)$	$r, x$ variation of perturbation pressure/ $(K_0 \rho a^2 \dot{\phi}^2)$
$\bar{p}_j(r)$	$j$ th radial eigenfunction in series given by the 3rd term in Eq. (2.6d)
$\hat{p}_0(r)$	first term in solution to $p$ , Eq. (2.6d)
$\hat{p}_k(r)$	coefficient of $\sin \lambda_k x$ in series given by the 2nd term of the right-hand side of Eq. (2.6d)
$p^*$	perturbation pressure/ $(K_0 \rho a^2 \dot{\phi}^2)$ , Eq. (2.1d)
$P$	axisymmetric unperturbed pressure/ $(\rho a^2 \dot{\phi}^2)$ , Wedemeyer spin-up model, Eq. (2.1d)
$r$	$(1/a) \times$ radial coordinate in inertial coordinate system
$R(r)$	error function defined in Eq. (2.16)
$Re$	Reynolds number = $a^2 \dot{\phi} / \nu$
$Re_E$	"effective" Reynolds number, Eq. (2.5)
$t$	time [s]
$u, v, w$	$(1/[a \dot{\phi}]) \times$ radial, azimuthal, axial velocity components, respectively, in inertial frame
$\underline{u}, \underline{v}, \underline{w}$	$(1/[K_0 a \dot{\phi}]) \times r, x$ variation of perturbation radial, azimuthal, axial velocity components, Eq. (2.2)
$\bar{u}_j, \bar{v}_j, \bar{w}_j$	coefficients of $\sin \mu_j x$ and $\cos \mu_j x$ in series expansions for $\underline{u}, \underline{v}, \underline{w}, p$ (3rd terms of right-hand sides of Eqs. (2.6))

# LIST OF SYMBOLS (Continued)

$\hat{u}_k, \hat{v}_k, \hat{w}_k$	coefficients of $\sin \lambda_k x$ and $\cos \lambda_k x$ in series expansions for $\underline{u}, \underline{v}, \underline{w}$ (2nd terms of right-hand sides of Eqs. (2.6))
$\hat{u}_0, \hat{v}_0, \hat{w}_0$	1st terms in right-hand sides of Eqs. (2.6a), (2.6b), and (2.6c)
$u^*, v^*, w^*$	$(1/[K_0 a \dot{\phi}]) \times$ radial, azimuthal, axial perturbation velocity components in inertial system, Eqs. (2.1) and (2.2)
$U, V, W$	$(1/[a \dot{\phi}]) \times$ radial, azimuthal, axial velocity components of axisymmetric unperturbed flow, Eq. (2.1)
$\hat{u}_i, \hat{v}_i, \hat{w}_i$	( $i = 1, 2, 3$ ) linearly independent solutions which are combined to obtain $\hat{u}_0, \hat{v}_0, \hat{w}_0$ , Eq. (2.12)
$x, y, z$	rectangular coordinates in inertial system (x-axis along trajectory) [length/a], $y = r \cos \theta$ , $z = r \sin \theta$
$\tilde{x}, \tilde{y}, \tilde{z}$	rectangular coordinates in aeroballistic system ( $\tilde{x}$ -axis along cylinder axis) [length/a]
$\beta_E$	quantity defined in Eq. (2.4b), used in evaluating $C_{(LSM)VE}$
$\delta C_E$	correction term in ad hoc endwall boundary condition, Eqs. (2.3) and (2.4c)
$\epsilon$	$= (1/\tau) \times$ yaw growth per radian of nutation
$\epsilon_p$	$= 0$ for $\lambda_k \neq 0$ , $= 1$ for $\lambda_k = 0$ (Eqs. (2.23) of Reference 1)
$\theta$	polar angle (azimuthal coordinate) in inertial system
$\lambda_k$	eigenvalue in axial problem, Eqs. (2.6) and (2.7)

# LIST OF SYMBOLS (Continued)

$\mu_j$	eigenvalues occurring in series given by the 3rd terms of the right-hand sides of Eq. (2.6); see paragraph preceding Eq. (2.8)
$\nu$	kinematic viscosity of liquid [cm <sup>2</sup> /s]
$\vec{\xi}$	vector describing angular motion of cylinder, Eqs. (1.1) and (1.2)
$\rho$	density of liquid [g/cm <sup>3</sup> ]
$\tau$	nutational frequency of cylinder/ $\dot{\phi}$
$\dot{\phi}$	spin rate of cylinder [rad/s], taken to be positive
$\psi(r)$	function defined in Eq. (2.17e), used in Eq. (2.16)

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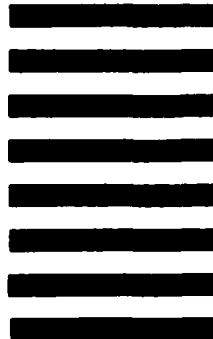


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